Basic Mathematics

Introductory Differentiation

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The aim of this document is to provide a short, self assessment programme for students who would like to acquire a basic understanding of elementary differentiation.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Rates of Change (Introduction)

Differentiation is concerned with the *rate of change* of one quantity with respect to another quantity.

Example 1

If a ball is thrown vertically upward with a speed of $10\,\mathrm{ms}^{-1}$ then the height of the ball, in metres, after t seconds is approximately

$$h(t) = 10t - 5t^2 \,.$$

Find the average speed of the ball during the following time intervals.

(a) from $t = 0.25\,\mathrm{s}$ to $t = 1\,\mathrm{s}$, (b) from $t = 0.25\,\mathrm{s}$ to $t = 0.5\,\mathrm{s}$.

Solution Average speed is

$$v_{\text{average}} = \frac{\text{distance travelled}}{\text{time taken}}$$

(a) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 1\,\mathrm{s}$ is

$$\begin{aligned} \frac{h(1) - h(0.25)}{1 - 0.25} &= \frac{(10 \times 1 - 5 \times 1^2) - (10 \times 0.25 - 5 \times 0.25^2)}{1 - 0.25} \\ &= \frac{5 - 2.1875}{0.75} = 3.75\,\mathrm{ms}^{-1}. \end{aligned}$$

(b) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 0.5\,\mathrm{s}$ is

$$\begin{aligned}\frac{h(0.5) - h(0.25)}{0.5 - 0.25} &= \frac{(10 \times 0.5 - 5 \times 0.5^2) - (10 \times 0.25 - 5 \times 0.25^2)}{0.5 - 0.25} \\ &= \frac{3.75 - 2.1875}{0.25} = 6.25\,\mathrm{ms}^{-1}.\end{aligned}$$

Exercise 1. Referring to **example 1**, find the average speed of the ball during the following time intervals. (Click on the green letters for solutions.)

(a) From $t = 0.25\,\mathrm{s}$ to $t = 0.375\,\mathrm{s}$, (b) From $t = 0.25\,\mathrm{s}$ to $t = 0.3125\,\mathrm{s}$,
(c) From $t = 0.25\,\mathrm{s}$ to $t = 0.251\,\mathrm{s}$, (d) From $t = 0.25\,\mathrm{s}$ to $t = 0.2501\,\mathrm{s}$.

Quiz Which of the following is a good choice for the *speed* of the ball when $t = 0.25\,\mathrm{s}$?

(a) $7.52\,\mathrm{ms}^{-1}$ (b) $7.50\,\mathrm{ms}^{-1}$ (c) $7.499\,\mathrm{ms}^{-1}$ (d) $7.49\,\mathrm{ms}^{-1}$

2. Rates of Change (Continued)

In the previous section the *speed* of the ball was found at $t = 0.25\,\mathrm{s}$. The next example gives the *general solution* to this problem.

Example 2

If, as in **example 1**, the height of a ball at time t is given by

$$h(t) = 10t - 5t^2\,, \qquad \text{then find the following :}$$

(a) the average speed of the ball over the time interval from t to $t+\delta t$,
(b) the *limit* of this average as $\delta t \to 0$.

Solution

(a)The height at time $t + \delta t$ is $h(t + \delta t)$ and the height at time t is $h(t)$. The difference in heights is $h(t+\delta t) - h(t)$ and the time interval is δt.

$$\begin{aligned} h(t + \delta t) - h(t) &= [10(t + \delta t) - 5(t + \delta t)^2] - [10t - 5t^2] \\ &= [10t + 10\delta t - 5(t^2 + 2t\delta t + (\delta t)^2] - [10t - 5t^2] \\ &= 10\delta t - 10t\delta t - 5(\delta t)^2 \\ &= \delta t[10 - 10t - 5\delta t]. \end{aligned}$$

The required average speed of the ball at time t is thus

$$\begin{aligned}\frac{h(t+\delta t)-h(t)}{\delta t} &= \frac{\delta t[10-10t-5\delta t]}{\delta t}\\ &= 10-10t-5\delta t\,,\end{aligned}$$

after cancelling the δt.

(b) As δt gets *smaller*, i.e. $\delta t \to 0$, the last term becomes negligible and the *instantaneous speed* at time t is $v(t)$, where

$$v(t) = 10 - 10t \quad \text{is the } \textit{speed} \text{ of the ball at time } t\,.$$

Exercise 2. Referring to the solution of **example 2**, find the speed of the particle when $t = 0.25\,\mathrm{s}$. (Click on exercise 2 for the solution.)

To recap, the speed $v(t)$ is obtained from the height $h(t)$ as

$$v(t) = \lim_{\delta t \to 0}\left[\frac{h(t+\delta t)-h(t)}{\delta t}\right].$$

3. The Derivative as a Limit

The diagram shows a function $y = f(x)$. The straight line AB has gradient BC/CA. As the point B moves along the curve toward A, the straight line AB tends toward the tangent to the curve at A. At the same time, the value of the gradient BC/CA tends toward the gradient of the tangent to the curve at A.

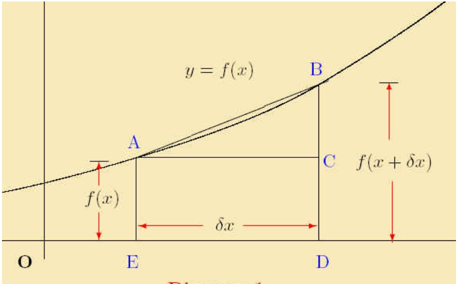

Diagram 1

From **diagram 1** $\quad \dfrac{\text{BC}}{\text{CA}} = \dfrac{f(x+\delta x) - f(x)}{\delta x}$.

Define $\quad \dfrac{dy}{dx} = \lim_{\delta x \to 0} \dfrac{f(x+\delta x) - f(x)}{\delta x}$.

The limit, dy/dx, is called the *derivative* of the function $f(x)$. Its value is the *gradient of the tangent* to the curve at the point A(x, y).

Example 3
Find the derivative of the function $y = x^3$.
Solution
For this problem $y = f(x) = x^3$ so the derivative is

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} \frac{(x+\delta x)^3 - x^3}{\delta x}$$

The numerator of this is

$$\begin{aligned} (x+\delta x)^3 - x^3 &= (x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3) - x^3 \\ &= 3x^2\delta x + 3x\delta x^2 + \delta x^3 \\ &= \delta x(3x^2 + 3x\delta x + \delta x^2)\,. \\ \text{Then} \qquad \frac{dy}{dx} &= \lim_{\delta x \to 0} \frac{\delta x(3x^2 + 3x\delta x + \delta x^2)}{\delta x} \\ &= \lim_{\delta x \to 0} (3x^2 + 3x\delta x + \delta x^2) \\ \text{The derivative is thus } \frac{dy}{dx} &= 3x^2\,. \end{aligned}$$

EXERCISE 3. For each of the following functions, use the technique of **example 3** to find the derivative of the function. (Click on the green letters for solutions.)

(a) $y = x$, (b) $y = x^2$, (c) $y = 1$.

Example 4

Find the gradient of the tangent to the curve $y = x^3$ at the point on the curve when $x = 2$.

Solution

From **example 3**, the derivative of this function is

$$\frac{dy}{dx} = 3x^2 \,.$$

The gradient of the tangent to the curve when $x = 2$ is

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 = 12 \,,$$

where the symbol $\left.\dfrac{dy}{dx}\right|_{x=2}$ is the function $\dfrac{dy}{dx}$ evaluated at $x = 2$.

EXERCISE 4. Find the gradient of the tangent to each of the following functions at the indicated points. (Click on green letters for solutions.)

(a) $y = x$ at the point with coordinates $(2, 2)$,
(b) $y = x^2$ at the point with coordinates $(3, 9)$,
(c) $y = 1$ at the point with coordinates $(27, 1)$.

Quiz Referring to **example 3** and exercise 3, which of the following is the most likely choice for the derivative of the function $y = x^4$?

(a) $4x^3$ (b) $3x^3$ (c) $4x^4$ (d) $3x^4$

Although the derivative of a function has been described in terms of a limiting process, it is not necessary to proceed in this fashion for each function. The derivatives for certain standard functions, and the rules of differentiation, are well known. The application of these rules, which is part of the discipline known as *calculus*, is the subject of the rest of this package.

4. Differentiation

The following table lists, *without proof*, the derivatives of some well-known functions. Throughout, a is a constant.

y	ax^n	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\ln(ax)$
$\dfrac{dy}{dx}$	nax^{n-1}	$a\cos(ax)$	$-a\sin(ax)$	$a\mathrm{e}^{ax}$	$\dfrac{1}{x}$

Here are two more useful *rules of differentiation*. They follow from the definition of differentiation but are stated *without proof*.
If a is any constant and u, v are two functions of x, then

$$\begin{aligned}\frac{d}{dx}(u+v) &= \frac{du}{dx}+\frac{dv}{dx}\\ \frac{d}{dx}(au) &= a\frac{du}{dx}\end{aligned}$$

The use of these rules is illustrated on the next page.

Example 5

For each of the following functions, find $\dfrac{dy}{dx}$.

$$\text{(a) } y = x^2 + 4x^3\,, \quad \text{(b) } y = 5x^2 + \frac{1}{x}\,, \quad \text{(c) } y = 5\sqrt{x} + \frac{3}{x^2} - 6x\,.$$

Solution

(a) Using the rules of differentiation

$$\begin{aligned} y &= x^2 + 4x^3 \\ \frac{dy}{dx} &= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^3) \\ &= 2x + 3 \times 4x^2 \;=\; 2x + 12x^2 \end{aligned}$$

(b) Before proceeding, note that $1/x = x^{-1}$ (see the package on **powers**). The function may now be written as

$$y = 5x^2 + \frac{1}{x} = 5x^2 + x^{-1}$$

and the rules can now be applied.

(b)(continued)

$$\begin{aligned} y &= 5x^2 + x^{-1} \\ \frac{dy}{dx} &= \frac{d}{dx}(5x^2) + \frac{d}{dx}(x^{-1}) \\ &= 2 \times 5x + (-1)x^{-2} \\ &= 10x - \frac{1}{x^2} \end{aligned}$$

(c) From the package on **powers**, $\sqrt{x} = x^{\frac{1}{2}}$, so

$$\begin{aligned} y &= 5x^{\frac{1}{2}} + 3x^{-2} - 6x \\ \frac{dy}{dx} &= \frac{d}{dx}(5x^{\frac{1}{2}}) + \frac{d}{dx}(3x^{-2}) - \frac{d}{dx}(6x) \\ &= \frac{1}{2} \times 5(x^{-\frac{1}{2}}) + (-2) \times 3(x^{-3}) - 6 \\ &= \frac{5}{2}x^{-\frac{1}{2}} - 6x^{-3} - 6 = \frac{5}{(2x^{\frac{1}{2}})} - \frac{6}{(x^3)} - 6 \end{aligned}$$

EXERCISE 5. Find dy/dx for each of the following functions. (Click on the green letters for solutions.)

(a) $y = 3x^4 + 4x^5$ (b) $y = 2\sqrt{x}$, (c) $y = \dfrac{4}{x^3} - 3\sqrt[3]{x}$.

Example 5 Find $\dfrac{dy}{dw}$ if $y = 2\sin(3w) - 3\cos(4w) + e^{4w}$.

Solution Using the rules

$$\begin{aligned}
\frac{dy}{dw} &= 2\frac{d}{dw}(\sin(3w)) - 3\frac{d}{dw}(\cos(4w)) + \frac{d}{dw}(e^{4w}) \\
&= 2(3\cos(3w)) - 3(-4\sin(4w)) + 4e^{4w} \\
&= 6\cos(3w) + 12\sin(4w) + 4e^{4w}
\end{aligned}$$

EXERCISE 6. Find the derivative with respect to z, i.e. dy/dz, of each of the following functions. (Click on the green letters for solutions.)

(a) $y = 2\sin\left(\frac{1}{2}z\right)$, (b) $y = \dfrac{4}{z} - 3\ln(4z)$,

(c) $y = 2\ln(7z) + 3\cos(2z)$, (d) $y = e^{3z} - 3e^{z}$.

5. Quiz on Differentiation

Begin Quiz Choose $\dfrac{dy}{dx}$ for each of the following functions.

1. $y = 4x^{-3} - 2\sin(x)$

(a) $-12x^{-2} - 2\cos(x)$, (b) $-12x^{-4} - 2\cos(x)$,
(c) $-12x^{-2} + 2\cos(x)$, (d) $-12x^{-4} + 2\cos(x)$.

2. $y = 3x^{\frac{1}{3}} + 4x^{-\frac{1}{4}}$

(a) $3x^{\frac{2}{3}} - 4x^{-\frac{5}{4}}$, (b) $x^{-\frac{2}{3}} - x^{-\frac{5}{4}}$,
(c) $9x^{\frac{2}{3}} - 4x^{-\frac{5}{4}}$, (d) $x^{-\frac{1}{3}} - x^{-\frac{5}{4}}$.

3. $y = 2\mathrm{e}^{-2x} + 5\ln(2x)$

(a) $\mathrm{e}^{-2x} + \dfrac{5}{x}$, (b) $\mathrm{e}^{-2x} + \dfrac{10}{x}$,
(c) $-4\mathrm{e}^{-2x} + \dfrac{5}{x}$, (d) $-4\mathrm{e}^{-2x} + \dfrac{10}{x}$.

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 0.375\,\mathrm{s}$ is

$$\begin{aligned} & \frac{h(0.375) - h(0.25)}{0.375 - 0.25} = \\ = \quad & \frac{(10 \times 0.375 - 5 \times 0.375^2) - (10 \times 0.25 - 5 \times 0.25^2)}{0.375 - 0.25} \\ = \quad & \frac{3.047 - 2.1875}{0.125} = 6.875\,\mathrm{ms}^{-1}. \end{aligned}$$

Click on the green square to return

□

Exercise 1(b) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 0.3125\,\mathrm{s}$ is

$$
\begin{aligned}
& \frac{h(0.3125) - h(0.25)}{0.3125 - 0.25} = \\
= \quad & \frac{(10 \times 0.3125 - 5 \times 0.3125^2) - (10 \times 0.25 - 5 \times 0.25^2)}{0.3125 - 0.25} \\
= \quad & \frac{3.047 - 2.1875}{0.0625} = 7.1875\,\mathrm{ms}^{-1}.
\end{aligned}
$$

Click on the green square to return

$\square$

Exercise 1(c) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 0.251\,\mathrm{s}$ is

$$
\begin{aligned}
& \frac{h(0.251) - h(0.25)}{0.251 - 0.25} = \\
= \quad & \frac{(10 \times 0.251 - 5 \times 0.251^2) - (10 \times 0.25 - 5 \times 0.25^2)}{0.251 - 0.25} \\
= \quad & \frac{3.047 - 2.1875}{0.001} = 7.495\,\mathrm{ms}^{-1}.
\end{aligned}
$$

Click on the green square to return $\square$

Exercise 1(d) The average speed from $t = 0.25\,\mathrm{s}$ to $t = 0.251\,\mathrm{s}$ is

$$
\begin{aligned}
& \frac{h(0.2501) - h(0.25)}{0.2501 - 0.25} = \\
= \quad & \frac{(10 \times 0.2501 - 5 \times 0.2501^2) - (10 \times 0.25 - 5 \times 0.25^2)}{0.2501 - 0.25} \\
= \quad & \frac{2.1882 - 2.1875}{0.0001} = 7.4995\,\mathrm{ms}^{-1}.
\end{aligned}
$$

Click on the green square to return □

Exercise 2.
The speed is found by putting $t = 0.25$ into $v(t) = 10 - 10t$. The resulting speed is

$$v(0.25) = 10 - 10 \times 0.25 = 10 - 2.5 = 7.5\,\mathrm{ms}^{-1}\,.$$

This was precisely the value chosen in the earlier **quiz**, confirming that the function $v(t) = 10 - 10t$ is indeed the speed of the ball at any time t.

Exercise 2

Exercise 3(a) The derivative of $y = f(x) = x$ is

$$\begin{aligned} \frac{dy}{dx} &= \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{(x+\delta x) - x}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{\delta x}{\delta x} = 1\,. \end{aligned}$$

The derivative is thus

$$\frac{dy}{dx} = 1\,.$$

This can also be deduced from the fact that $y = x$ represents a straight line with gradient 1.

Click on the green square to return

$\square$

Exercise 3(b) The derivative of $y = f(x) = x^2$ is

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \\
&= \lim_{\delta x \to 0} \frac{(x+\delta x)^2 - x^2}{\delta x} \\
&= \lim_{\delta x \to 0} \frac{x^2 + 2x\delta x + (\delta x)^2 - x^2}{\delta x} \\
&= \lim_{\delta x \to 0} \frac{2x\delta x + (\delta x)^2}{\delta x} \\
&= \lim_{\delta x \to 0} \frac{(2x+\delta x)\delta x}{\delta x} \\
&= \lim_{\delta x \to 0} (2x + \delta x) = 2x\,.
\end{aligned}$$

The derivative is thus

$$\frac{dy}{dx} = 2x\,.$$

Click on the green square to return $\square$

Exercise 3(c) The derivative of $y = f(x) = 1$ is

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{1-1}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{0}{\delta x} \\ &= \lim_{\delta x \to 0} (0) = 0\,.\end{aligned}$$

The derivative of $y = 1$ is thus

$$\frac{dy}{dx} = 0\,.$$

This is a special case of the rule that the derivative of a constant is always zero.

Click on the green square to return □

Exercise 4(a) According to **exercise 3** the derivative of the function $y = x$ is

$$\frac{dy}{dx} = 1\,.$$

Therefore the gradient of the tangent to the curve at the point with coordinates $(2, 2)$, i.e. when $x = 2$, is

$$\left.\frac{dy}{dx}\right|_{x=2} = 1\,.$$

This is the gradient of the straight line $y = x$.
Click on the green square to return

□

Exercise 4(b) From **exercise 3** the derivative of the function $y = x^2$ is

$$\frac{dy}{dx} = 2x\,.$$

Therefore the gradient of the tangent to the curve at the point with coordinates $(3, 9)$ is the value of $\dfrac{dy}{dx}$ at $x = 3$, i.e.

$$\left.\frac{dy}{dx}\right|_{x=3} = 2 \times 3 = 6\,.$$

Click on the green square to return

□

Exercise 4(c) From **exercise 3** the derivative of the constant function $y = 1$ is

$$\frac{dy}{dx} = 0\,.$$

Therefore the gradient of the tangent to the curve at any point, including the point with coordinates $(27, 1)$, is zero, i.e.

$$\left.\frac{dy}{dx}\right|_{x=3} = 0\,.$$

Click on the green square to return $\square$

Exercise 5(a) Using the rules of differentiation

$$
\begin{aligned}
y &= 3x^4 + 4x^5 \\
\frac{dy}{dx} &= \frac{d}{dx}(3x^4) + \frac{d}{dx}(4x^5) \\
&= 4 \times 3x^{(4-1)} + 5 \times 4x^{(5-1)}) \\
&= 4 \times 3x^3 + 5 \times 4x^4 = 12x^3 + 20x^4
\end{aligned}
$$

Click on the green square to return $\square$

Exercise 5(b) From the package on **powers**, $2\sqrt{x} = 2x^{\frac{1}{2}}$, so using the rules of differentiation

$$
\begin{aligned}
y &= 2x^{\frac{1}{2}} \\
\frac{dy}{dx} &= \frac{d}{dx}(2x^{\frac{1}{2}}) \\
&= \frac{1}{2} \times 2x^{(\frac{1}{2}-1)} \\
&= x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}
\end{aligned}
$$

Click on the green square to return

$\square$

Exercise 5(c) The function may be rewritten (see the package on **powers**) as,

$$y = \frac{4}{x^3} - 3\sqrt[3]{x} = 4x^{-3} - 3x^{\frac{1}{3}},$$

and using the rules of differentiation

$$\begin{aligned}
y &= 4x^{-3} - 3x^{\frac{1}{3}} \\
\frac{dy}{dx} &= \frac{d}{dx}(4x^{-3}) - \frac{d}{dx}(3x^{\frac{1}{3}}) \\
&= (-3) \times 4x^{(-3-1)} - (\frac{1}{3}) \times 3\ x^{(\frac{1}{3}-1)} \\
&= -12x^{-4} - x^{-\frac{2}{3}} = -\frac{12}{x^4} - \frac{1}{x^{\frac{2}{3}}} \\
&= -\frac{12}{x^4} - \frac{1}{\sqrt[3]{x^2}}\,.
\end{aligned}$$

Click on the green square to return □

Exercise 6(a) Using the rules of differentiation and the table of derivatives

$$\begin{aligned}\frac{dy}{dz} &= 2\frac{d}{dz}\left(\sin\left(\frac{1}{2}z\right)\right)\\ &= 2\times\frac{1}{2}\cos\left(\frac{1}{2}z\right)\\ &= \sin\left(\frac{1}{2}z\right)\end{aligned}$$

Click on the green square to return

□

Exercise 6(b) Rewriting the function $y = \dfrac{4}{z} - 3\ln(4z)$ as

$$y = 4z^{-1} - 3\ln(4z)$$

and using the table of derivatives

$$\begin{aligned} \frac{dy}{dz} &= \frac{d}{dz}(4z^{-1}) - 3\frac{d}{dz}(\ln(4z)) \\ &= (-1) \times 4z^{(-1-1)} - 3 \times \frac{1}{z} \\ &= -4z^{-2} - \frac{3}{z} = -\frac{4}{z^2} - \frac{3}{z} \end{aligned}$$

Click on the green square to return

$\square$

Exercise 6(c) Using the rules of differentiation and the table of derivatives

$$
\begin{aligned}
\frac{dy}{dz} &= 2\frac{d}{dz}(\,\ln(7z)) + 3\frac{d}{dz}(\,\cos(2z)) \\
&= 2 \times \frac{1}{z} + 3 \times (-2\sin(2z)) \\
&= \frac{2}{z} - 6\sin 2z
\end{aligned}
$$

Click on the green square to return $\square$

Exercise 6(d) Since $\frac{d}{dz}\left(\mathrm{e}^{az}\right) = a\mathrm{e}^{az}$,

$$\begin{aligned} \frac{dy}{dz} &= \frac{d}{dz}\left(\mathrm{e}^{3z}\right) - 3\frac{d}{dz}\left(\mathrm{e}^{z}\right) \\ &= 3\mathrm{e}^{3z} - 3\mathrm{e}^{z} \\ &= 3(\mathrm{e}^{3z} - \mathrm{e}^{z})\,. \end{aligned}$$

Click on the green square to return $\square$

Solutions to Quizzes

Solution to Quiz: The table below shows the details of the calculations that were done in **example 1** and exercise 1.

Times distance measured (s)	Time interval (s)	Average speed (ms^{-1})
$t = 0.25$ to $t = 1$	0.75	3.75
$t = 0.25$ to $t = 0.5$	0.25	6.25
$t = 0.25$ to $t = 0.375$	0.125	6.87
$t = 0.25$ to $t = 0.3125$	0.0625	7.1875
$t = 0.25$ to $t = 0.251$	0.001	7.495
$t = 0.25$ to $t = 0.2501$	0.0001	7.4995

The difference in speeds is measured over decreasing intervals of time staring at $t = 0.25\,\mathrm{s}$. As this interval of time decreases, so the average speed tends towards $7.5\,\mathrm{ms}^{-1}$. This is then taken to be the *speed* of the ball at the point when $t = 0.25\,\mathrm{s}$. This *limiting process*, taking averages over smaller and smaller intervals, is at the heart of differentiation.

End Quiz

Solution to Quiz: The table below shows the details of the calculations that were done in **example 3** and exercise 3.

Function	Derivative
$y = x^3$	$\dfrac{dy}{dx} = 3x^2$
$y = x^2$	$\dfrac{dy}{dx} = 2x \ (= 2x^1 = 2x^{2-1})$
$y = x \ (= x^1)$	$\dfrac{dy}{dx} = 1 \ (= x^0 = 1x^{1-1})$
$y = 1 \ (= x^0)$	$\dfrac{dy}{dx} = 0 \ (= 0x^{0-1})$

The general form, which is given without proof, is:

$$\text{if } y = x^n \text{ then } \frac{dy}{dx} = nx^{n-1}\,.$$

Thus if $y = x^4$ then $\dfrac{dy}{dx} = 4x^3$. End Quiz